High precision scale setting

Z. Fodor

University of Wuppertal & Budapest, UCSD, FZJ

Budapest-Marseille-Wuppertal Collaboration, 1203.4469*

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Outline

Scale settings and the static potential

raw output of lattice QCD: physical quantities in lattice unit
$\Rightarrow$ measure a dimensionful quantity Q (M_Ω or f_K)
the lattice spacing is given by a=$(aQ^{lat})/Q^{exp}$

today erros below 2% for several lattice predictions
it depends crucially on the error of the lattice spacing
need for a controlled/small error lattice spacing determination

not necessarily directly accesable for experiments e.g. potential
popular choices are:
string tension (strictly speaking doesn't exist: string breaking)
the Sommer-scale $r_x^2 \cdot dV/dr = C_x$
originally r_0 with $C_0 = 1.65$ or MILC choice r_1 with C_1=1

Sommer-scale, Omega mass, f_π and f_K

unfortunately, the calculations of r_0 & r_1 are quite involved
far more complicated than fitting the masses of particles

complications are reflected in the literature
MILC: r_1=0.3117(22) fm (less than 1% accuracy)
RBC/UKQCD: r_1=0.3333(93)(1)(2) fm
7% difference and 2.3σ tension between them

another popular way is to use the Omega baryon mass
the experimental value of M_Ω is well known
more CPU demanding & sensititve to the strange quark mass
mismatched strange quark mass leads to a mismatched scale

difficulties with f_π (chiral extrapolation) & f_K (mismatched m_s)

suggestion of M. Luscher: use the Wilson flow to set the scale

Definition of the flow of the gauge field

Morningstar, Peardon PRD 69 (2004) 054501; Narayan, Neuberger, JHEP 0603 (2006) 064; Luscher JHEP 1008 (2010) 071

consider the flow: $B_\mu(t,x)$ for $t>0$ with $B_\mu(0,x)=A_\mu(x)$
flow equation: $\partial_t B_\mu = D_\nu G_{\mu\nu}$ with $G_{\mu\nu}=\partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$

the evolution in t has a smoothing effect:

$\partial_t B_\mu = \Delta B_\mu - \partial_\mu \partial_\nu B_\nu$ + non-linear terms

the first term is the same as in the heat-equation

$B_\mu(t,x) = \int d^4x K_t(x-y) A_\mu(y) + ...$

K_t four dimensional heat kernel $K_t(r)=\exp(-r^2/4t)/(4\pi t)^2$
smoothing effect with $\sqrt{8t}$ smoothing range

on the lattice regularize it: $V_t(x,\mu)$ for $t>0$ with $V_0(x,\mu)=U(x,\mu)$
flow equation with (Z) staples: $\partial_t V_t(x,\mu) = Z(V_t(x,\mu)) \cdot V_t(x,\mu)$

Wilson flow: technical realization

flow equation: $\dot{V}_t = Z(V_t)V_t$, where Z is the staple
equivalent to a series of infinitesimal stout smearing steps

in our case it is integrated with 4th-order Runge-Kutta scheme

M. Luscher, JHEP 1008 (2010) 071

evolution from time t to time $t+\epsilon$ is given by $Z_i = \epsilon Z(W_i)$

$$
\begin{aligned}
W_0 &= V_t, \\
W_1 &= \exp\left(\frac{1}{4}Z_0\right) W_0, \\
W_2 &= \exp\left(\frac{8}{9}Z_1 - \frac{17}{36}Z_0\right) W_1, \\
V_{t+\epsilon} &= \exp\left(\frac{3}{4}Z_2 - \frac{8}{9}Z_1 + \frac{17}{36}Z_0\right) W_2
\end{aligned}
$$

Wilson flow and the coupling

M. Luscher, JHEP 1008 (2010) 071

as a representative example $E = G^a_{\mu\nu} G^a_{\mu\nu}/4$ is considered
lattice: E(t) can be defined by the (1-plaquette) or clover terms
they only differ by discretization effects
lattice: we expect $\langle E \rangle \propto$(1-plaquette)$\cdot t^2$ behavior

very important results about the renormalization of the Wilson flow

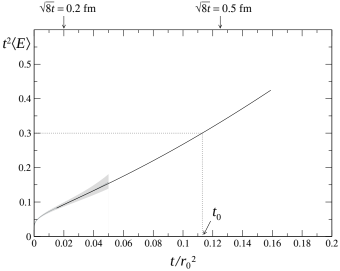
calculation of $\langle E \rangle$ up to $\alpha_s^2(q)$ with $q = (8t)^{-1/2}$
(result has been obtained in the continuum $\overline{MS}$ scheme)

$$\langle E \rangle = \frac{3}{4\pi t^2} \alpha(q) \{1 + k_1 \alpha(q) + \mathcal{O}(\alpha^2)\}, \quad k_1 = 1.0978 + 0.0075 N_f$$

above the cut-off (small t): lattice and continuum quite different

Lattice study of the Wilson flow (pure gauge)

the perturbation QCD expansion works for small t ($\ll$1 fm)
for large t one uses numerical lattice simulations
SU(3) pure gauge theory with lattice spacing a=0.05 fm

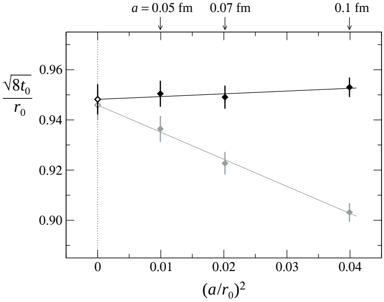


statistical error: smaller than the thickness of the (linear) line
lattice: expect (1-plaquette)$\cdot t^2$ behavior for small t
perturbation theory is given by the band (uncertainty on Λ)

Wilson flow for scale setting: quenched

$\langle E \rangle$ is physical: approaches its continuum limit with a^2
test it with the reference scale t_0 given by

$$\left\{ t^2 \langle E \rangle \right\}_{t=t_0} = 0.3$$

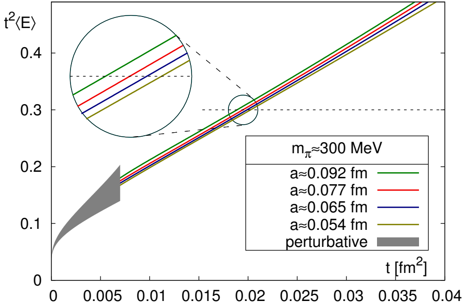


scaling violation increases toward smaller reference scales
for which the smoothing range is only 2-3 times the lattice spacing

Gauge flow for dynamical fermions & w_0

one can determine the gauge flow also for the dynamical case
use the Wilson flow or the gauge flow defined by the action

$t^2\langle E(t)\rangle$ incorporates informations from all $t{>}\mathcal{O}(1/\sqrt{t})$
its derivative (almost constant) mostly from scales around $\mathcal{O}(1/\sqrt{t})$
advantage: flow at small $t \sim a^2$ is a subject of cutoff effects

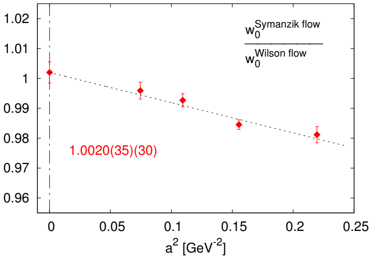


observed "linearity" for $t^2\langle E\rangle$
one can extract it by $t \cdot dt^2\langle E\rangle/dt$
instead $t^2\langle E\rangle{=}0.3$ (M. Luscher)
$t \cdot dt^2\langle E\rangle/dt{=}0.3$ (w_0 scale)

a$\rightarrow$0: non-universal part shrinks
w_0 has less cutoff effects than t_0

Continuum limit is the same

different definitions should have the same continuum limit
one can use the Wilson flow or the Symanzik flow: M_π=135 MeV



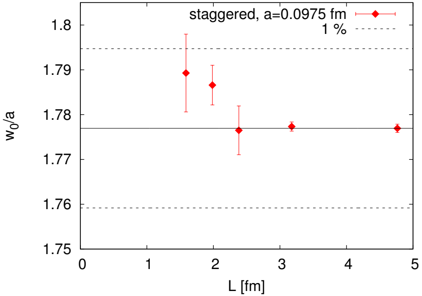
original definition of Luscher has the largest cut-off effect
various definitions of w_0 have tiny ones (a few % or less)
(statistical errors are neglible, good for scale setting)

Finite volume effects

how sensitive is w_0 to the size of the system

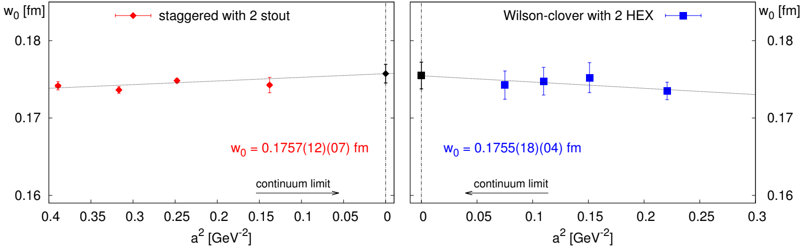
only for boxes $<$2 fm: $M_\pi L \approx 1.35$ instead of 4

$\Longrightarrow$ finite volume effects are tiny, far below the 1% level



robust and stable method for determining the scale

a $\longrightarrow$ 0: Wilson & staggered w_0 Budapest-Marseille-Wuppertal Collaboration, 1203.4469*



the physical scale was obtained by the Omega baryon mass

our final result is the Wilson result (staggered is a cross check)
(no rooting $\Longrightarrow$ theoretically cleaner)

w_0=0.1755(18)(04) fm

error (dominantly statistical) is 1%
(and comes not from the gauge flow itself, but from M_Ω)

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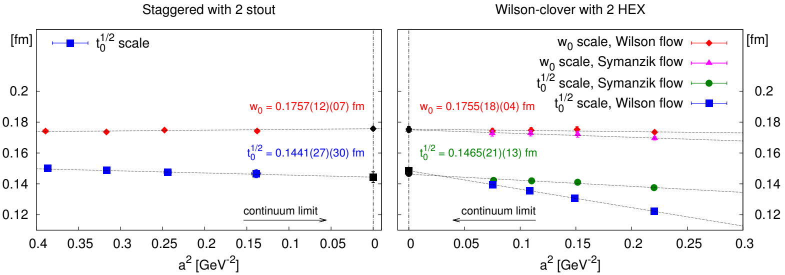

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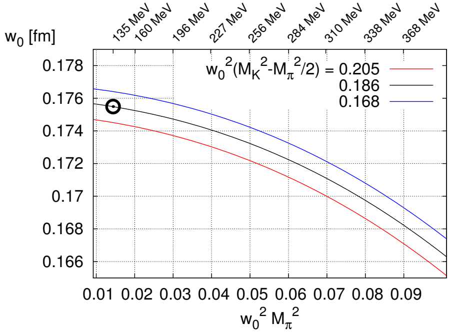
Scale off the physical point

usually runs aren't at physical masses: what is the scale there
measure M_π, M_K and w_0: $x = w_0^2 M_\pi^2$ and $y = w_0^2(M_K^2 - M_\pi^2/2)$

w_0=0.18515-0.5885x^2-0.0497y-0.11xy-1.476x^3$\pm$18$\cdot$10^{-3}$\pm$4$\cdot$10^{-3}[fm]



change M_π from 135 to 350 MeV

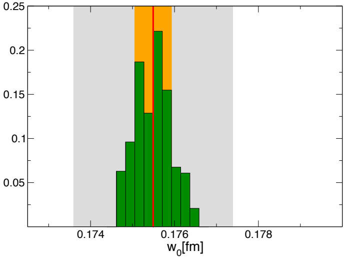
4% change in the lattice spacing
(same size as cutoff effects)

change m_s by 10%
0.5% change in the lattice spacing

error is 1% in the continuum limit

Error analysis: 2HEX data set

histogram method to give statistical and systematic errors
64 possible results (m_q interpolation, M_π cut, a→0, fit range, scale)



orange/gray bands: systematic/full error; red line: result

interpolation	M_π-cut	a→0	fit range	scale
15%	40%	55%	55%	45%

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